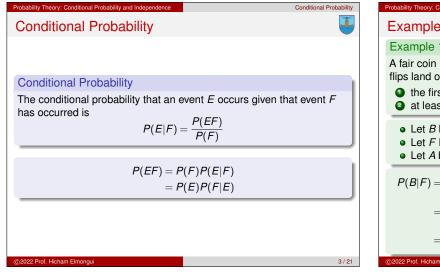
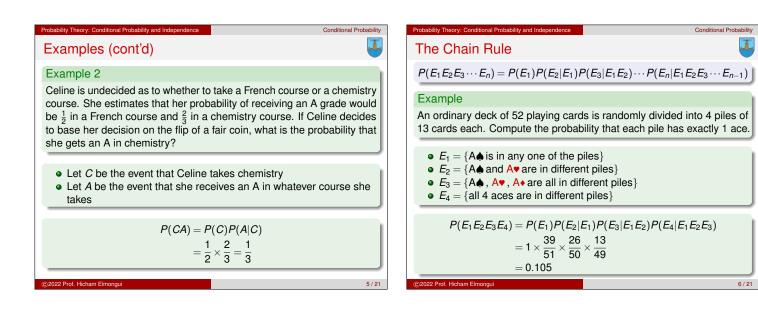
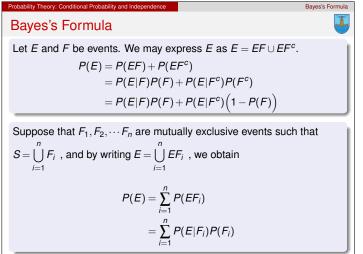


• The desired probability is $\frac{1}{6}$



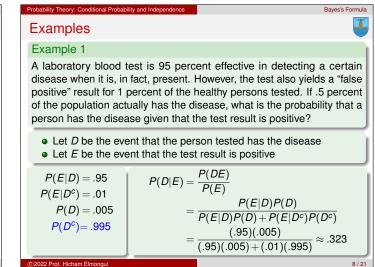
Conditional Probability and IndependenceConditional Probability Theory: Conditional ProbabilityExamplesExample 1A fair coin is flipped twice. What is the conditional probability that both flips land on heads, given thatImage: the first flip lands on heads?Image: the first flip lands on heads?Image: the last one flip lands?Image: the last one flip last one flip lands?Image: the last one flip last one flip last one flip last one heads?Image: the last one flip last one flip last one flip last one flip last one heads?Ima





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Probability Theory: Conditional Probabi	ity and Independence	Bayes's Formula
Examples		
Example 1 (cont'd)		
What is the probatest result is negative	bility that a person has the disease given?	n that the
$P(E D) = .95$ $P(E D^{c}) = .01$ $P(D) = .005$ $P(D^{c}) = .995$ $P(E^{c} D^{c}) = .99$ $P(E^{c} D) = .05$	$P(D E^{c}) = \frac{P(DE^{c})}{P(E^{c})}$ $= \frac{P(E^{c} D)P(D)}{P(E^{c} D)P(D) + P(E^{c} D^{c})}$ $= \frac{(.05)(.005)}{(.05)(.005) + (.99)(.995)}$ ≈ 0.00025	P(D°)
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Probability Theory: Conditional Probability and Independence	Bayes's Formula
Examples (cont'd)	4
Example 2	
A new couple, known to have two children, has just moved Suppose that the mother is encountered walking with one of h If this child is a girl, what is the probability that both children	ner children.
 G₁ ≜ event that the first child is a girl (B₁ ≜ boy) G₂ ≜ event that the second child is a girl (B₂ ≜ boy) G ≜ event that the child seen with the mother is a girl (B₂ ≤ boy) 	B≜ boy)
$P(G_1 G_2 G) = rac{P(G_1 G_2 G)}{P(G)} = rac{P(G_1 G_2)}{P(G)}$	
$P(G) = P(G G_1G_2)P(G_1G_2) + P(G G_1B_2)P(G_1B_2)$	
$+ P(G B_1G_2)P(B_1G_2) + P(G B_1B_2)P(B_1B_2)$	
$= P(G_1G_2) + P(G G_1B_2)P(G_1B_2) + P(G B_1G_2)P(G_1B_2) + P(G B_1G_2)P(G_1B_2)P(G_1B_2) + P(G B_1G_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_1B_2)P(G_2)P(G_2)P(G_2)P(G_2)P(G_2)P(G_2)P(G_2)P(G_2)P(G_2)P(G_2)P(G_2)P(G_2)P(G_2)P(G_2)P(G_2)P(G_2)P(G_2)P(G_2)P(G_2)P(G_2)P(G_2)P(G_2)P(G_2)P(G_2)P(G_2)P(G_2)P(G_2)P(G_2)P(G_2)P(G_2)P(G_2)P(G_2)P(G_2)P(G_2)P(G_2)$	(B_1G_2)
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Potobality target count dependenceExpansion (Count dependence)Examples (cont'd)Image: cont dependenceExamples (cont'd)Image: cont dependence
$$P(G_1G_2|G) = \frac{P(G_1G_2) + P(G|G_1B_2)P(G_1B_2) + P(G|B_1G_2) + P(G|B_1G_2) + \frac{1}{4} + \frac{1}{4}P(G|G_1B_2) + \frac{1}{4}P(G|B_1G_2) + \frac{1}{4} + \frac{1}{4}P(G|G_1B_2) + \frac{1}{4}P(G|B_1G_2) + \frac{1}{4}P(G|B_1G_2) + \frac{1}{4} + \frac{1}{4}P(G|G_1B_2) + \frac{1}{4}P(G|B_1G_2) + \frac{1}{4}P(G|B_1G_2)$$

Probability Theory: Conditional Probability and Independence	Bayes's Formula	
Odds of an Event	—	
Definition		
The odds of an event A tell how mu occurs than it is that it does not occur		
$O(A) = rac{P(A)}{P(A^c)} = rac{P(A)}{1 - P(A)}$	$P(A) = \frac{O(A)}{1 + O(A)}$	
if $P(A) = \frac{2}{3}$, then $P(A) = 2P(A^c)$, so the odds are 2 to 1 in favor of A		
Consider a hypothesis <i>H</i> that is true that new evidence <i>E</i> is introduced	with probability $P(H)$, and suppose	
$P(H E) = rac{P(E H)P(H)}{P(E)}$	$P(H^c E) = rac{P(E H^c)P(H^c)}{P(E)}$	
$O(H E) = O(H) imes rac{P(E H)}{P(E H^c)}$		
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Example

Example

An urn contains two type *A* coins and one type *B* coin. When a type *A* coin is flipped, it comes up heads with probability 1/4, whereas when a type *B* coin is flipped, it comes up heads with probability 3/4. A coin is randomly chosen from the urn and flipped. Given that the flip landed on heads, what is the probability that it was a type *A* coin?

• Let *A* be the event that a type *A* coin was flipped
• Therefore
$$A^c$$
 is the event that a type *B* coin was flipped
 $O(A|\text{heads}) = O(A) \times \frac{P(\text{heads}|A)}{P(A|\text{heads})} = \frac{O(A|\text{heads})}{P(A|\text{heads})}$

Independent Events

Definition

Bayes's Formula

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E is independent of F if knowledge that F has occurred does not change the probability that E occurs.

Independen

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$$P(E|F) = P(E)$$

Since P(E|F) = P(EF)/P(F), it follows that *E* is independent of *F* if P(EF) = P(E)P(F)

The fact that the last equation is symmetric in E and F shows that whenever E is independent of F, F is also independent of E.

Two events *E* and *F* that are not independent are said to be dependent.

If *E* and *F* are independent events, then so are *E* and *F*^c.

$$P(E) = P(EF) + P(EF^{c}) = P(E)P(F) + P(EF^{c})$$

$$P(EF^{c}) = P(E)(1 - P(F)) = P(E)P(F^{c})$$

Probability Theory: Conditional Probability and Independence	Independent Events	
Example	<u> </u>	
Example		
Suppose that we toss 2 fair dice.		
• Let <i>E</i> denote the event that the first die equals 4.		
• Let <i>F</i> ₁ denote the event that the sum of the dice is 6		
• Let F_2 denote the event that the sum of the dice is 7		
Are E and F_1 independent? What about E and F_2 ?		
$P(EF_1) = P(\{(4,2)\}) = \frac{1}{36}$ $P(E)P(F_1) = \frac{1}{6} \times \frac{5}{36} = \frac{5}{216}$ $\neq P(EF_1)$	$P(EF_2) = P(\{(4,3)\}) = \frac{1}{36}$ $P(E)P(F_2) = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$ $= P(EF_2)$	
E and F_1 are not independent	E and F_2 are independent	
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Probability Theory: Conditional Probability and Independence	Independent Events
Independence of a Set of Events	1
Three events <i>E</i> , <i>F</i> , and <i>G</i> are said to be independent if P(EFG) = P(E)P(F)P(G) $P(EF) = P(E)P(F)$ $P(EG) = P(E)P(G)$ $P(FG) = P(F)P(G)$	
The events E_1, E_2, \dots, E_n are said to be independent if, for $E_{1'}, E_{2'}, \dots, E_{r'}, r' \le n$ of these events $P(E_{1'}, E_{2'}, \dots, E_{r'}) = P(E_{1'})P(E_{2'}) \cdots P(E_{r'})$	every subset
An infinite set of events are defined to be independent subset of those events is independent.	if every finite

Probability Theory: Conditional Probability and Independence	Independent Events	
Examples	—	
Example		
Independent trials consisting of rolling a pair of fair dice are performed. What is the probability that an outcome of 5 appears before an outcome of 7 when the outcome of a roll is the sum of the dice?		
$E_n \triangleq$ event that no 5 or 7 appears on the first $n-1$ trials and a 5 appears on the n^{th} trial		
Since $P\{5 \text{ on any trial}\} = \frac{4}{36}$ and $P\{7 \text{ on any trial}\} = \frac{6}{36}$, we obtain, by the independence of trials $P(E_n) = \left(1 - \frac{10}{36}\right)^{n-1} \times \frac{4}{36} \qquad P\left(\bigcup_{n=1}^{\infty} E_n\right) = \sum_{n=1}^{\infty} P(E_n)$ $= \frac{1}{9} \left(\frac{13}{18}\right)^{n-1} \qquad = \frac{1}{9} \sum_{n=1}^{\infty} \left(\frac{13}{18}\right)^{n-1}$		
9(18)	$= \frac{1}{9} \sum_{n=1}^{\infty} \left(\frac{13}{18}\right)^{n-1}$ $= \frac{1}{9} \times \frac{1}{1 - \frac{13}{18}} = \frac{2}{5}$	
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robability Theory: Conditional Probability and Independence	P(. F) is a Probability
P(. F) is a Probability	—
Conditional probabilities satisfy all of the properties of ordinary probabilities • $0 \le P(E F) \le 1$ • $P(S F) = 1$ • For any sequence of mutually exclusive events $E_1, E_2, \cdots, P\left(\bigcup_{i=1}^{\infty} E_i F\right) = \sum_{i=1}^{\infty} P(E_i F)$	
If we define $Q(E) \triangleq P(E F)$, then $Q(E)$ may be regarded as a probability function on the events of <i>S</i> . Hence, all of the propositions previously proved for probabilities apply to $Q(E)$. For instance,	
$Q(E_1 \cup E_2) = Q(E_1) + Q(E_2) - Q(E_1E_2)$ $P(E_1 \cup E_2 F) = P(E_1 F) + P(E_2 F) - P(E_1E_2 F)$)

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P(.|F) is a Probability (cont'd)

 $Q(E_1E_2E_3\cdots E_n) = Q(E_1)Q(E_2|E_1)Q(E_3|E_1E_2)\cdots Q(E_n|E_1E_2E_3\cdots E_{n-1})$ $P(E_1E_2E_3\cdots E_n|F) = P(E_1|F)P(E_2|E_1F)P(E_3|E_1E_2F)\cdots$ $P(E_n|E_1E_2E_3\cdots E_{n-1}F)$

If we define $Q(E_1|E_2) \triangleq Q(E_1E_2)/Q(E_2)$, then

 $\begin{aligned} Q(E_1) &= Q(E_1|E_2)Q(E_2) + Q(E_1|E_2^c)Q(E_2^c) \\ P(E_1|F) &= P(E_1|E_2F)P(E_2|F) + P(E_1|E_2^cF)P(E_2^c|F) \end{aligned}$

We say that the events E_1 and E_2 are *conditionally independent* given F if, given that F occurs, the conditional probability that E_1 occurs is unchanged by information as to whether or not E_2 occurs $P(E_1|E_2F) = P(E_1|F)$

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 $P(E_1E_2|F) = P(E_1|F)P(E_2|F)$

Probability Theory: Conditional Probability and Independence Example (cont'd)	P(. F) is a Probability
 Let <i>M_i</i> be the event that male #<i>i</i>, <i>i</i> = 1,2, is Let <i>B_{A,a}</i> be the event that the baby chimp 	
$P(M_1 B_{A,a}) = \frac{P(M_1B_{A,a})}{P(B_{A,a})}$ $= \frac{P(B_{A,a} M_1)P}{P(B_{A,a} M_1)P(M_1) + P(M_1)}$ $= \frac{1 \times p}{1 \times p + 1/2 \times (1-p)}$ $= \frac{2p}{1+p} > p \qquad (p < p)$ The information that the baby's gene pair is (Ability that male #1 is the father.	B _{A,a} M ₂)P(M ₂) 1)

Example

P(.|F) is a Probability

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Example

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A female chimp has given birth. It is not certain, however, which of two male chimps is the father. Before any genetic analysis has been performed, it is felt that the probability that male #1 is the father is p and the probability that male #2 is the father is 1 - p. DNA obtained from the mother, male #1, and male #2 indicate that, on one specific location of the genome, the mother has the gene pair (A, A), male #1 has the gene pair (a, a), and male #2 has the gene pair (A, a). If a DNA test shows that the baby chimp has the gene pair (A, a), what is the probability that male #1 is the father?

P(.|F) is a Pr

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Let all probabilities be conditional on the event that the mother has the gene pair (A, A), male #1 has the gene pair (a, a), and male #2 has the gene pair (A, a).